Indian Statistical Institute, Bangalore Centre

B.Math (Hons.) III Year, First Semester

Semestral Examination - 2013-2014

Complex Analysis

Time: 2.00pm to 5.00pm November 8, 2013 Instructor: Bhaskar Bagchi

Full Marks : 100.

1. (a) Let g be a non-constant holomorphic function, and f be a continuous function on the image of g. Then show that f is holomorphic iff $f\ o\ g$ is holomorphic by establishing the chain rule.

   (b) If a holomorphic function has a continuous branch of its logarithm then show that this branch is holomorphic.

   (c) If a holomorphic function has an inverse then show that the inverse is holomorphic.

   [12+4+4=20]

2. (a) If a holomorphic function is injective then show that it has no essential isolated singularity.

   (b) If an entire function is injective then show that it is of the form $z \longmapsto az + l$ for constants a, l.

   [10+10=20]

3. (a) If a holomorphic function has an everywhere non-vanishing derivative then show that it is a local homeomorphism.

   (b) Show that the limit of a locally uniformly convergent sequence of injective holomorphic functions is either injective or constant. Show by examples that both alternatives can occur.

   [12+8=20]

4. (a) Define the Mobius maps on the unit disc and show that they form a group under composition.

   (b) Show that the group in part (a) is the full group of biholomorphic automorphisms of the unit disc.

   [12+8=20]

5. Let $A = \{z : r < \mid z \mid < R\}$ where $0 \leq r < R$. Let f be a holomorphic function on A. Then show that there is a Laurent series which represents f throughout A. [20]